

## Numerical Techniques for Underwater Topographic Measurement with GPS and Echo Sounder

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### Abstract

*This study demonstrates numerical techniques for a topographic study of a water area. Data obtained by a GPS unit and an echo sounder were combined, and three dimensional data were generated. A problem to find a surface that contains those data points was reformulated to a problem to find a piecewise linear continuous function defined over a triangular mesh. An iterative method led to an approximation of a fixed point. Those techniques are illustrated, and numerical examples are introduced.*

**Keywords:** Water area topography, Triangular mesh, Fixed point iteration, GPS, Echo sounder.

### 1. Introduction

Seto Inland Sea is an inland sea between the main island and the Shikoku Island of Japan. Kojima Bay is located in the coast of the Seto Inland Sea along the main island. A part of the Kojima Bay was closed by an embankment to be a reservoir named Kojima Lake. The area of the Kojima Lake is approximately 10 km<sup>2</sup>. Inflows from two rivers, Sasagase river and Kurashiki river are primary source of the water in the Kojima Lake. There are six gates set in the bank between the Kojima Lake and the Kojima Bay. The water level of the Kojima Lake is maintained by the discharge through the gates into the Kojima Bay. The topography of such a reservoir is subject to change due to erosion and sedimentation, and updated topographic data should be provided on a regular basis. Topographic surveys of the Kojima Lake were conducted on September 28<sup>th</sup> and October 4<sup>th</sup> in 2019. Positional data were recorded by a real time kinematic GPS (RTK-GPS) unit with a virtual reference station (VRS) system, while depth data were recorded by an echo sounder mounted on a vessel (Figure 1).



A GPS antenna attached to one end of a pole (a)



An oscillator of an echo sounder was attached to the other end of the pole beneath the water surface (b)

**Figure 1. RTK-GPS and echo sounder**

Positional data on the reference ellipsoid expressed in terms of longitude and latitude were transformed to rectangular coordinates with the Gauss-Krüger Projection. Those data were combined with the depth data to be three dimensional topographic data. Those data obtained from the topographic measurement were introduced into a topographic analysis. A triangular mesh in the plane was set. The problem to find a surface that contains the measurement data was reformulated into a fixed point problem of a map on values on nodes in a triangular mesh. Numerical techniques are illustrated, and numerical results are introduced.

## 2. Research Methodology

### 2.1. RTK-GPS Data and Depth Data

Topographic data of the Kojima Lake were obtained from the measurement conducted on September 28<sup>th</sup> and October 4<sup>th</sup>. The positional data were obtained by the RTK-GPS. Ellipsoidal coordinates given in terms of the longitude and the latitude were transformed to rectangular coordinates with the Gauss-Krüger Projection. Figure 2 shows the RTK-GPS tracks in the plane.

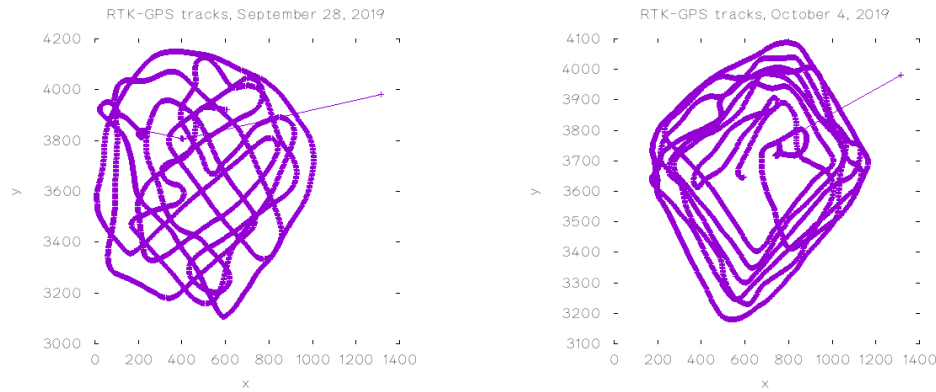
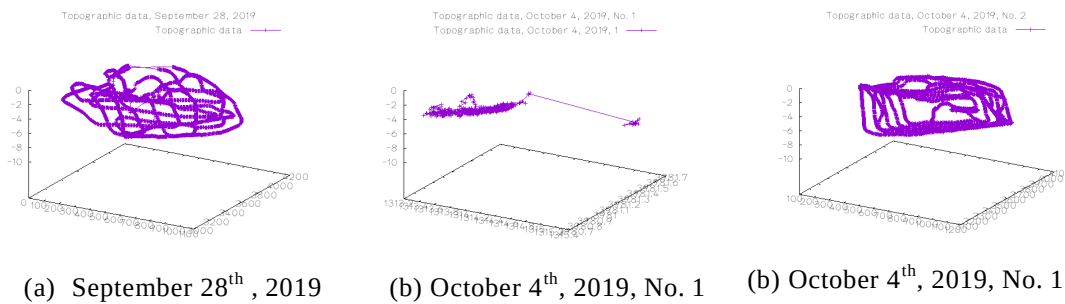


Figure 2. RTK-GPS tracks

The RTK-GPS data were combined with the depth data recorded by the echo sounder to be three dimensional topographic data were generated (Figure 3). The  $z$  component of the topographic data  $(x_j, y_j, z_j)$  was defined by

$$z_j = h_j - d_j - z_0 - l, \quad (1)$$

where  $h_j$  [m] is the antenna height,  $d_j$  [m] is the output from the echo sounder,  $z_0$  [m] ( $z_0 = 36.652$ ) is the geodetic height of the mean sea level, and  $l$  [m] ( $l = 2.315$ ) is the distance between the antenna and the oscillator.



(a) September 28<sup>th</sup>, 2019

(b) October 4<sup>th</sup>, 2019, No. 1

(b) October 4<sup>th</sup>, 2019, No. 1

Figure 3. Three dimensional topographic data.

### B. Formulation of Map of Node Data and Fixed Point Iteration

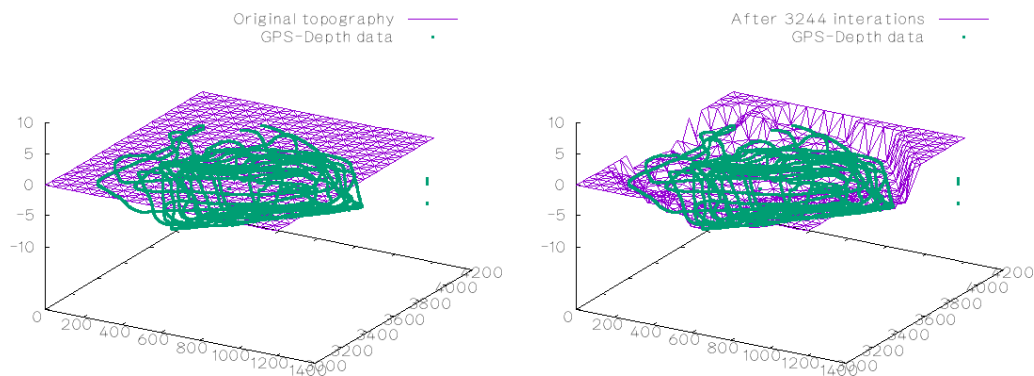
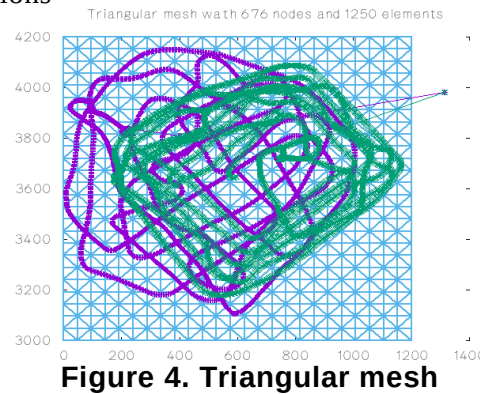
Let  $m$  be the number of elements and  $n$  be the number of nodes in a triangular mesh in the plane. Figure 4 shows a triangular mesh over a region that contains all the significant GPS tracks. Consider a topography expressed by a graph of a continuous function of  $x$  and  $y$ ,  $z = f(x,y)$ , whose domain is the region over the triangular mesh. Consider the vector space of piece wise linear continuous functions that are linear in each of elements. Suppose that the element  $k$  contains  $(x,y)$  components of the  $l$  data points,  $(x_1, y_1, z_1), (x_2, y_2, z_2), \dots, (x_l, y_l, z_l)$ . Suppose that those data points include

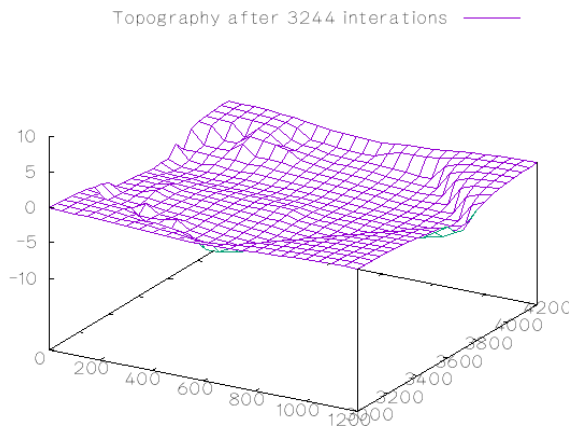
$$(x_{k,1}, y_{k,1}, z_{k,1}), (x_{k,2}, y_{k,2}, z_{k,2}), (x_{k,3}, y_{k,3}, z_{k,3}) \quad (2)$$

where  $(x_{k,1}, y_{k,1}), (x_{k,2}, y_{k,2}), (x_{k,3}, y_{k,3})$  are the vertices of the element  $k$ . The values of coefficients  $a$ ,  $b$ , and  $c$  of the linear function  $z = ax + by + c$  for the element  $k$  were those that minimize the square sum

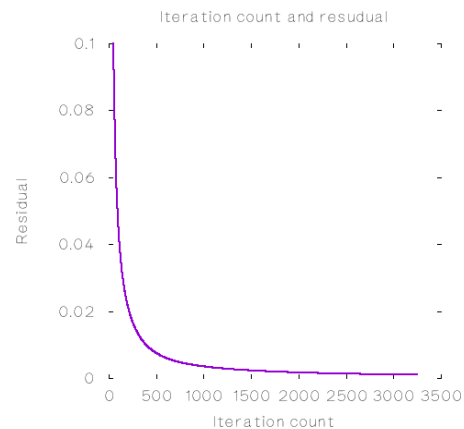
$$\sum_{j=1}^l [z_j - (ax_j + by_j + c)]^2 \quad (3)$$

Once those coefficients were determined for all the elements, the value of the  $z$  components at each node was determined as the area weighted average of all the values of linear functions over all the elements surrounding it. Thus a map on the  $n$  dimensional space was defined. It took 3244 fixed point iterations till the residual error reduced to 0.001. Figures 5 and 6 show numerical results. Figure 7 shows the transition of residuals, that is, the transition of the norm of the distance between two successive approximations





**Figure 6. Surface after 3244 fixed point iterations.**



**Figure 7. Transition of residuals with iteration count.**

### 3. Results and Discussion

Figure 5 and 6 show that our numerical results are acceptable for practical applications. Recent disastrous heavy rain events resulted in a number of casualties and tremendous damages to infrastructures and houses. Damages of the 2018 Japan floods amounted to 119 people killed, 29 people wounded, 213 houses completely destroyed, 340 houses half destroyed, and 290 houses partially destroyed [1]. The 2019 Typhoon 19 caused damages including 95 people killed, 5 people missing, 40 people severely wounded, 435 people slightly wounded, 1981 houses completely destroyed, 10168 houses half destroyed, and 12510 houses partially destroyed [1]. Those disastrous events remind us about importance of preventive maintenance of rivers, reservoirs, and coastal areas as well as measurement and analytical systems for underwater topography.

### 4. Conclusion

Techniques to update the underwater topography using RTK-GPS data and echo sounder data were previously developed [3]. In this paper, the fixed point iteration of depth data at the node is described

### Acknowledgement

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